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PROFES PROBABILISITC FINITE ELEMENT SYSTEM -- BRINGING PROBABILISTIC MECHANICS TO THE DESKTOP

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ABSTRACT

ProFES is a probabilistic finite element analysis system that allows designers to perform probabilistic finite element analysis in a 3D graphical environment that is completely familiar and similar to modern deterministic FEA. ProFES is built on an innovative data-driven software architecture that seamlessly integrates state-of-the-art probabilistic mechanics techniques with commercial CAD and CAE software to make it practical and feasible to execute probabilistic analysis of complex structural components. This capability allows engineers to use powerful probabilistic techniques to solve challenging problems such as probabilistic high cycle fatigue (HCF) analysis without the need for extensive training in probabilistic mechanics techniques. Industry partners in the ProFES development include: GE, Pratt&Whitney, Allison Engine, General Motors and commercial FEA vendors ANSYS and MSC/NASTRAN. This paper describes ProFES and probabilistic mechanics and shows three demonstration problems run using ProFES.

INTRODUCTION

As international competition drives industry to produce more reliable and affordable products that can be brought to market faster, there is a greater need for robust designs that balance performance and reliability. Major corporations are turning to probabilistic methods. Probabilistic methods provide a means to assess the affect of design uncertainties and manufacturing tolerances, to predict product reliability and performance, and achieve this optimal balance of performance, cost, and reliability.

INTRODUCTION TO PROBABILISTIC MECHANICS

Analysts and designers are confronted with numerous uncertainties and product variability. They must consider manufacturing tolerances, loads, material properties, and boundary conditions, and must design with these uncertainties in mind to ensure that products are reliable and safe. To address these variables in traditional (deterministic) design, we apply safety factors. The reliability-based design approach uses probabilistic methods to take a more structured view and explicitly models these uncertainties as random variables.

This article provides background on reliability-based design and introduces a new tool called ProFES. The simple examples described herein provide a brief introduction to the ProFES tool. The references that we have cited present the details of probabilistic methods and their applications.

Using ProFES, designs can now be optimized for performance and reliability. ProFES works with commercial finite element codes, such as ANSYS and MSC/NASTRAN, to simplify the modeling of uncertainties and to evaluate the affect of these uncertainties on product reliability. ProFES can also be used to determine optimum manufacturing tolerances.

Through advances in probabilistic mechanics, analysts can now solve very complex problems that were impractical until recently. A comprehensive review of the state-of-the-art in probabilistic mechanics (including probabilistic finite elements, random fields, probabilistic fracture mechanics and life prediction) can be found in the recent 30-chapter handbook compiled by Sundararajan [1995];[13] and the recent text by Ditlevsen and Madsen [1996].[6] Other well-known texts are Madsen, et al. [1986];[9] Ang and Tang [1984][2]; and Benjamin and Cornell [1970].[4] Detailed reviews of individual probabilistic methods include an excellent review of the most-probable-point approaches by Schueller [1989];[12] a discussion of adaptive response surface approaches by Rajashekhar and Ellingwood [1993];[10] and a review and comparison of adaptive importance sampling methods by Engelund and Rackwitz [1993].[7] Some recent applications in industry include reviews of research on probabilistic design of gas turbine engines by Adamson [1996][1] and Roth [1996];[11] and in the automotive industry by Twu, et al. [1998].[14] Now these tools must be placed in the hands of practitioners and must be usable without extensive

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re-training and software development efforts. The next few sections describe the steps in a reliability-based analysis and provide short examples executed using ProFES and ANSYS.

IDENTIFYING AND QUANTIFYING SOURCES OF UNCERTAINTY

In reliability-based analysis, uncertainties in numerical values are modeled as random variables. Quantities typically modeled as random include loads, material properties, element properties, boundary conditions, dimensions, and finite element model discretization error. A reliability-based analysis is needed if one or more quantity is modeled as a random variable. In a simple load uncertainty example, the applied mechanical force at a particular node is uncertain or the temperatures along a boundary are uncertain. Other examples of uncertainty are:

- Material property variability (e.g., in modulus of elasticity, Poisson's ratio, or yield strength).
- Element properties, as exemplified by the cross-sectional properties of a beam element.
- Boundary conditions (e.g., degree of fixity of a support, the precise location of an attachment point).
- Structure dimensions, as exemplified by the radius of a curve on a machined part.

The degree of variability depends on the type of material and the quality control in the manufacturing process. For example, variability may be much higher for a composite than a more traditional material, such as a common aluminum alloy.

Finally, finite element models are only approximations of the true structure, so there is inevitably some error associated with their use. The modeling error, in itself, represents an additional source of uncertainty.

Each random variable is assigned a probability distribution. In general, the distribution can be defined by a mean, μ, a standard deviation, σ, and a distribution type. Note that some distributions will require bounds or additional shape parameters. If you have many data points, the distribution may be found from data by standard statistical methods. However, frequently the analyst only has a rough notion of the mean and perhaps the standard deviation. If the analyst can make statements of the form "The mean force is 100 lbs. and there is a 5% probability that the force is greater than 120 lbs.," then the statistics can be estimated. Even though these statistics must be estimated, this approach provides more accurate results than using an arbitrary load factor or knockdown factor. Also, in such cases, the predicted reliability or probability of failure is necessarily approximate; however, the probabilistic approach will still provide very useful information in the form of sensitivity factors.

The sensitivity factors, which are a by-product of the probabilistic analysis, tell the analyst which sources of uncertainty are contributing most to the uncertainty in the predicted performance. Thus, the analyst can determine which variables should be better controlled to attain the best improvement in product reliability. Alternatively, the analyst can determine which tolerances could be relaxed without substantially affecting product reliability.

DEFINING FAILURE MODES

Failure modes in probabilistic analysis are called limit-states and are modeled using a limit-state equation (sometimes called a performance function). The most common form of the limit-state is $g=R-S$ where R is a resistance or capacity and S is a load effect. Examples of a limit-state are the stress at a particular location (S) exceeding the yield stress (R), or the calculated number of cycles to fatigue failure (R) totaling less than the desired product life (S). When the limit-state function evaluates to less than zero, the system is considered to have failed.

SELECTING PROBABILISTIC ANALYSIS METHOD

A number of methods are available to speed the probabilistic analysis and minimize the number of finite element computations required. Essentially all methods involve repeated evaluation of the limit-state function, which in many cases will require repeated finite element computations.

When selecting the probabilistic analysis method for a given problem the analyst must consider the following criteria:

- How long does a deterministic analysis take?
- What failure probability is expected?
- How many random variables does the problem have?
- What computational resources are available?

In certain limited cases analytic solutions are possible for simple limit-states. For example, if all random variables are normally distributed and if the limit-state is a linear function of the random variables, then the exact answer is available in closed-form.

In general, however, a numerical approach must be used. Three basic methods for numerical probabilistic analysis are Monte Carlo simulation (MCS), first-order-second-moment analysis (FOSM), and the first-order-reliability-method (FORM). Advanced methods include adaptive response surface approaches (RS) and adaptive importance sampling (IS).

- ***MCS.*** MCS generates samples of each random variable, and runs the deterministic model at each combination. Statistics and probabilities are determined by a simple statistical analysis of the results.
- ***FOSM.*** FOSM finds the gradients of the limit state function at the mean values of the random variables, fits at linear response surface at this point, and estimates mean and standard deviation of response.
- ***FORM.*** FORM searches the input variables for the combination that is most likely to cause failure (this point is often referred to as the *design point* or the *most probable point* (MPP)). It then fits a linear surface at the MPP and uses this surface (along with transformations for any non-normal random variables) to compute probabilities. A variant of this method is **SORM**; wherein a second order surface is fit at the MPP.
- ***RS.*** The RS method fits a surface to the response quantity (usually by first sampling the response using Design of Experiments techniques) then uses MCS or IS on the surface to perform the probabilistic analysis. The surface can be refit in critical areas of the response to improve the accuracy of the results (e.g., about the MPP).
- ***IS.*** The IS approach samples random variables near the critical regions so that better failure probability estimates can be obtained with less samples than MCS.

MCS is best for problems that execute relatively quickly (e.g., simple linear FE models or closed form expressions) because it typically requires a large number of limit state function evaluations (the number of evaluations is a function of the probability of failure). FOSM is good for estimating mean and variance of response if the response is not a highly nonlinear function of the random variables; but, in general, is not appropriate for estimating probabilities. FORM is preferred for evaluating small probabilities because it often requires the least number of finite element analyses. For FORM, the computational effort is proportional to the number of random variables. Advanced methods can result in improved probability estimates with little or no increase in computational effort. In general, the best method for any problem, in terms of computational efficiency and accuracy, will depend on the number of random variables, the number of different limit state functions, the degree of nonlinearity in the limit state function, and the probability of failure.

PROBABILISTIC ANALYSIS RESULTS

Using probabilistic analysis methods you can calculate the probability of failure events, as well as means, and standard deviations of response. Additionally, these methods provide sensitivity measures. The results of a FORM analysis, for example, include so-called α values for each input random variable. The α value squared is a measure of the variance in the response that is due to variance in the input. Methods are also available for finding derivatives of the response mean or failure probability w.r.t. input variable distribution parameters (mean, etc.)

EXAMPLE PROBLEMS

Example 1 – Plate-with-Hole

Figure 1 shows a simple model of a plate with a hole. The ANSYS model used to solve the deterministic problem has 398 nodes and 162 triangular elements. The load, the modulus of elasticity, and Poisson's ratio are modeled as random variables. The random variable distributions are shown in Table 1. The plate is loaded in uniform tension along the far right hand edge and supported at the opposite left hand edge. The finite element result of interest is the stress near the circumference of the hole.

Table 1. Input random variables for plate-with-hole finite element model.

Parameter	Dist	Mean	Std.Dev.	Units
Modulus	Lognormal	2.9×10^7	2.9×10^6	psi
Poisson's Ratio	Truncated Lognormal	0.25	0.025	--
Load	Lognormal	1.0	0.1	lb

ProFES was used to specify which quantities in the model are random, define the distributions of the random variables, define response variables, define a limit-state, execute a Monte Carlo simulation and FORM analysis, and view the results. Figures 2 through 4 picture some of these steps along with the results.

Figure 2 illustrates how we specify that the modulus of elasticity is random and define its statistics and distribution. We first add *modulus of elasticity* to the list of random variables by selecting it from the *Material Properties Dialog Box* (ProFES automatically

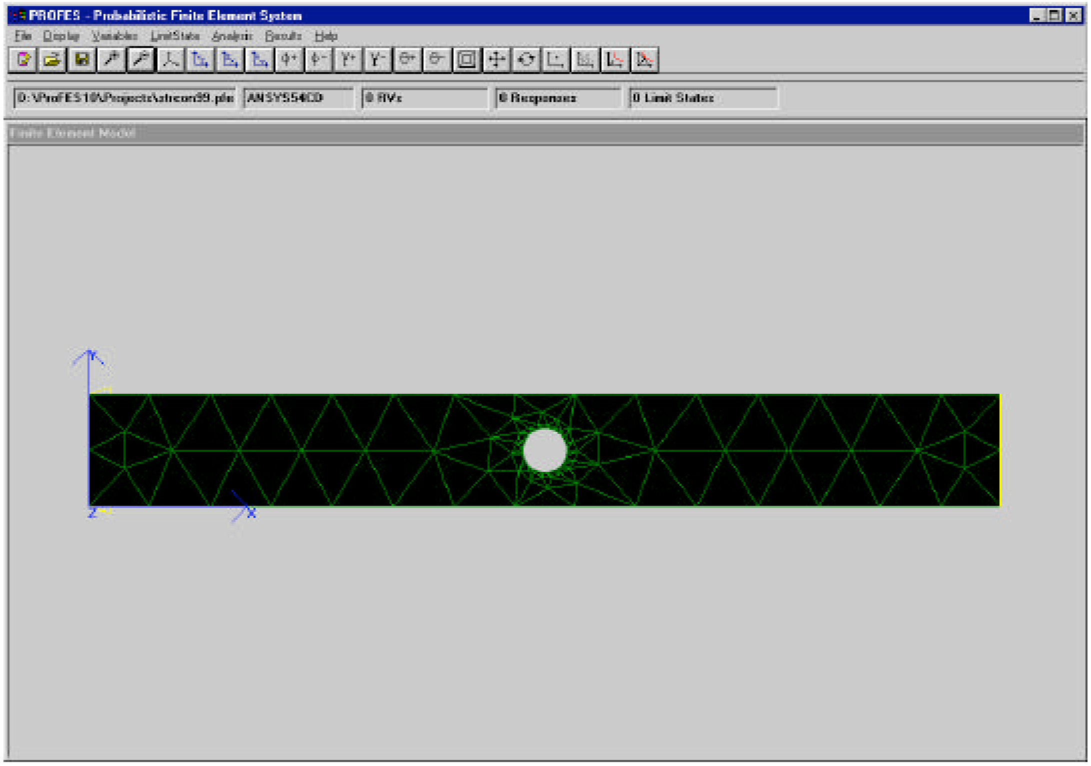

Figure 1. An ANSYS model of a plate with a central hole is imported into the ProFES application directly from an ANSYS database or input file.

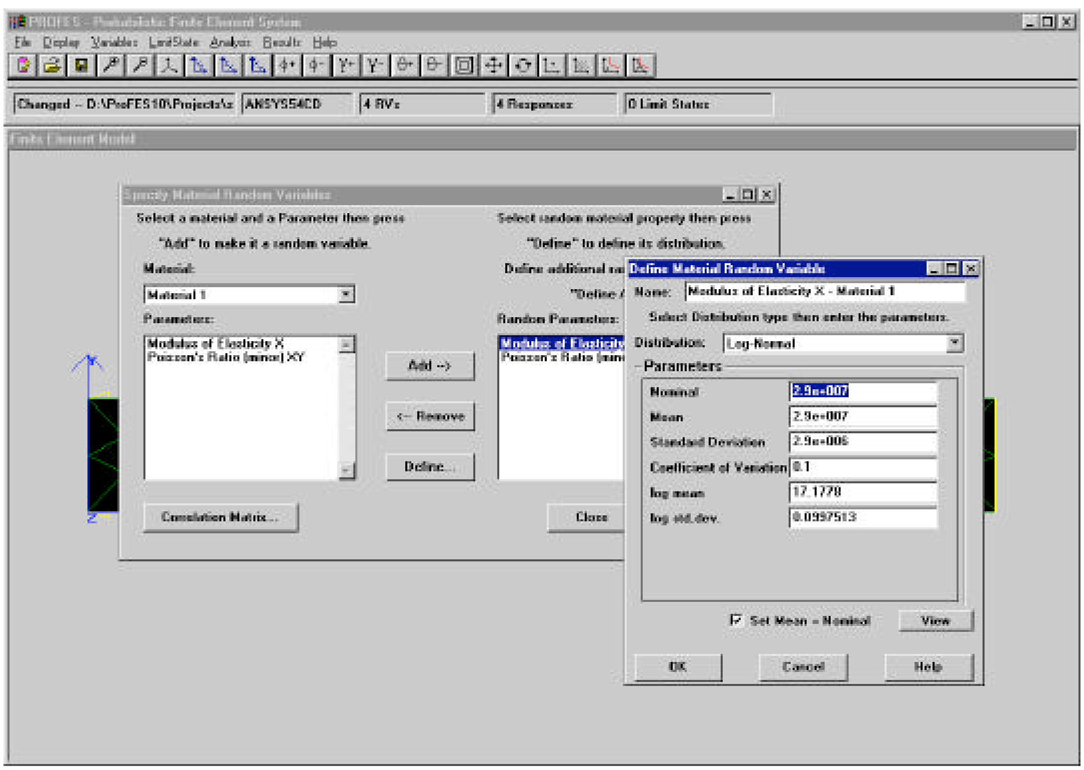

Figure 2. The first step in the probabilistic analysis is to specify the quantities that are random variables and define their distributions.

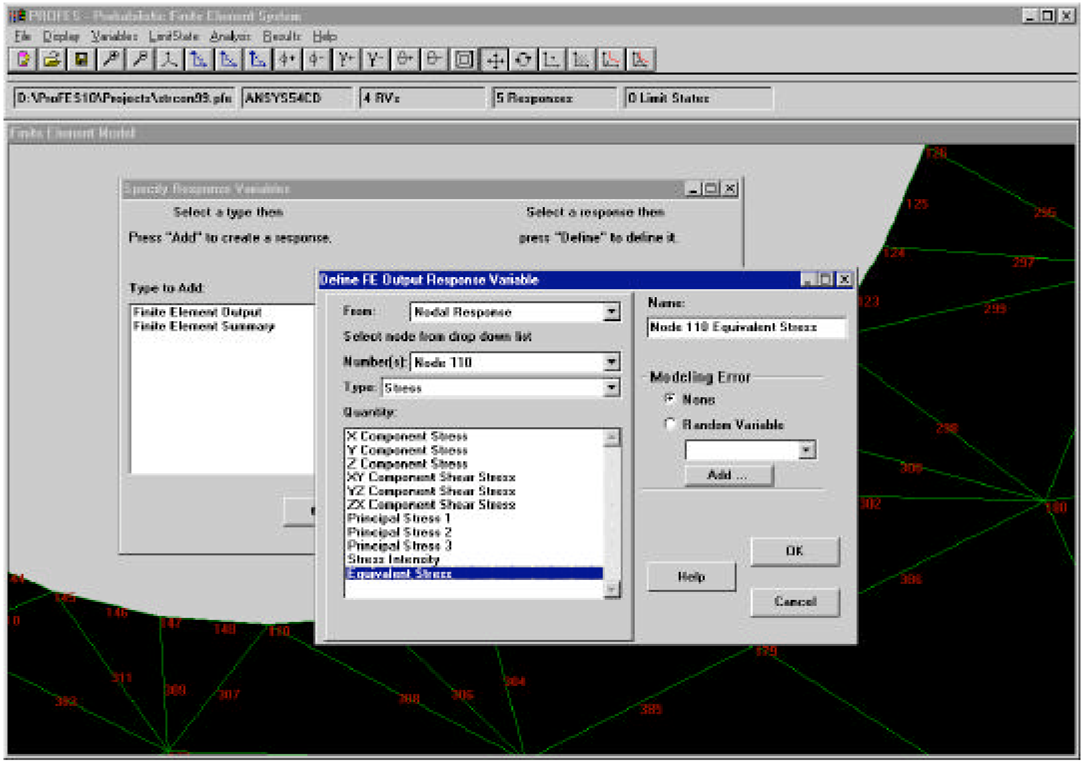

Figure 3. Response variables are selected from the ANSYS results and may be used in limit-state functions. The user has zoomed in to select critical nodes

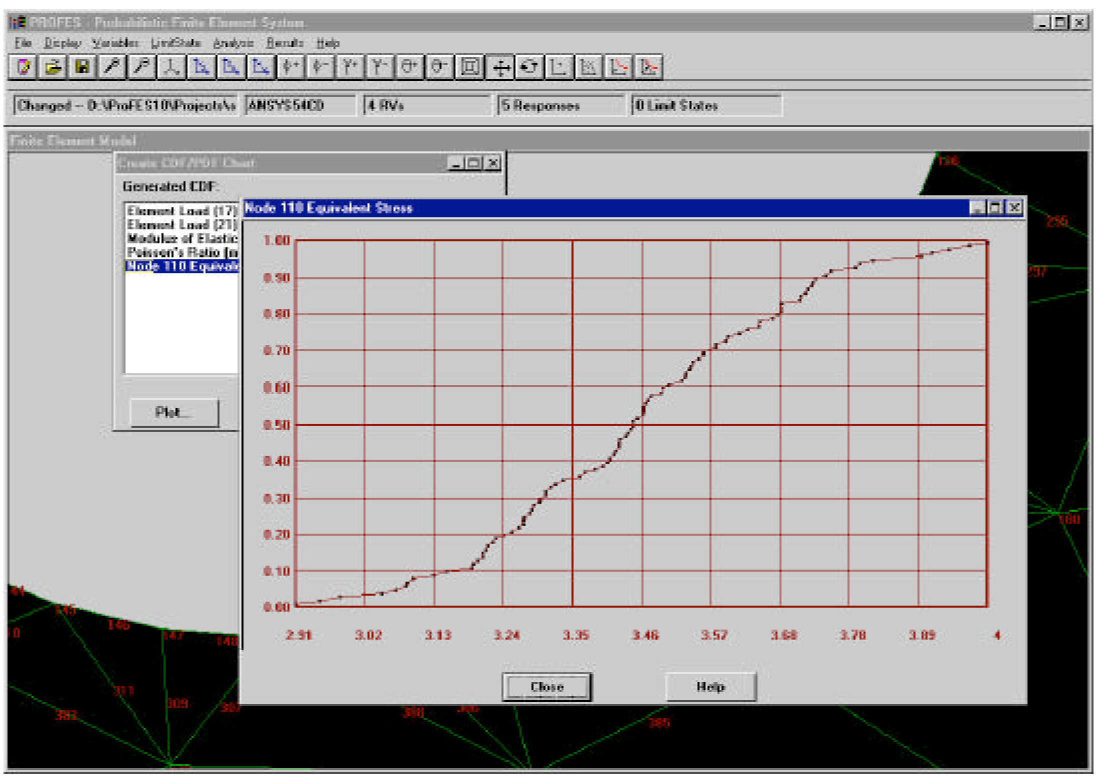

Figure 4. Probabilistic analysis results can be presented in the form of a cumulative distribution function. The cumulative distribution function is the probability, shown on the y-axis, that the stress concentration is less than the x-axis value. The results can also be viewed as a histogram or in report form.

identifies all the material properties in the finite element model and presents them to the user). Next, we define the probability distribution for the modulus using the *Define Material Random Variable Dialog Box*. Within the dialog box you can name the random quantity, select the type of probability distribution (from a list of nine different distribution types), and define this distribution in terms of any combination of its parameters. There is also a button that allows you to view the distribution graphically.

After defining the random input variables, we specify which response variables are to be collected by ProFES for statistical analysis or for use in a limit state function. In Figure 3 we select the equivalent stress at node 110. Any response quantity available from the finite element analysis can be selected (ProFES presents users with a list of all possible response quantities, e.g. displacements, stresses, strains, etc.).

The problem specification is now complete and we are ready to perform probabilistic analysis on the finite element model. The current version of ProFES includes First Order Second Moment (FOSM), First Order Reliability Method (FORM), Response Surface, and Monte Carlo Simulation (MCS) (see description of methods provided earlier). Implementation of adaptive response surface and importance sampling methods is in progress. For this first illustration we use the MCS method by selecting Monte Carlo from the analysis menu. The cumulative distribution function shown in Figure 4 was generated by the MCS. The user can select any of the random or response quantities and generate CDF/PDF curves for each. The cumulative distribution is the probability, shown on the y-axis, that the stress concentration is less than the x-axis value.

The results shown in the CDF of Figure 4 can be interpreted as a stress concentration factor because the far field stress is 1.0 in this finite element model. We found that the stress concentration factor has a mean value of 3.45 and a standard deviation of 0.23. One of the samples had a stress greater than four.

<u>Example 2 – Plate-with-Hole/Parametric Model</u>

ProFES also has the ability to import parametric models from ANSYS. Parametric models are useful for modeling geometric uncertainties that arise due to manufacturing tolerances. In order to evaluate uncertainty in the stress due to uncertainty in the radius of the hole, the parametric mode in ProFES was used to reanalyze this component. In addition to the three uncertainties in Table 1, the parameter R, which represents the hole radius in the ANSYS parametric model, was specified to be a random variable and defined to have a normal distribution, with a mean of 0.5 and standard deviation of 0.05. In the parametric mode ProFES utilizes ANSYS' scripting language to generate a solid model and finite element model based on the parameter, R. Hence, we can perform the finite element analysis for different values of R without any user intervention.

For this problem the plate will fail if the stress concentration exceeds 4.0. To evaluate the plate reliability (which is one minus the probability of failure) we instruct ProFES to compute the probability that the stress concentration exceeds 4.0 by defining a limit state function. Figure 5 shows the ProFES *Define Limit State Dialog Box*. The user has assigned the left-hand-side of the limit state function to the stress at node 110 (selected by pressing the *Define LHS* button and selecting the stress from the dialogue shown in Figure 3) and the right-hand-side to 4.0. For this analysis we used the First Order Reliability Method to analyze the limit state. The *Run First Order Reliability Analysis Dialog Box* (selected from the analysis menu) is shown in Figure 6. Here we simply select *Run*, since the default values work best in most cases. Advanced users can fine tune the analysis, however, using the options.

Figure 7 shows the results of the probabilistic analysis and the relative importance of each of the four random variables. For this problem the probability of failure, P_f, (probability that the stress concentration exceeds 4.0) is shown to be 0.131. The reliability is 1-0.131 = 0.869. The P_f value of 0.131 is relatively high and implies that the component will have a 13.1% chance of failing (stress concentrates exceeds 4.0).

How does the probabilistic result compare to a traditional deterministic analysis? A deterministic analysis using only mean values will indicate that the plate would not fail since the mean stress concentration is approximately 3.5. However, due to the problem uncertainties, the probabilistic analysis shows that there is a chance (13.1%) that the plate will fail. If this probability is deemed too high, the component would be redesigned. In a deterministic design we use safety factors to account for these uncertainties. The safety margin here is 4/3.5 = 1.14. It would be up to the designer to apply experience or use an experience-based code to determine if the safety margin is adequate. Most likely, this safety margin would be deemed inadequate and the component would be redesigned. However, with the safety factor approach, the designer has no knowledge as to whether the component is under or over-designed with regard to its actual probability of failure, or reliability. Also, for new

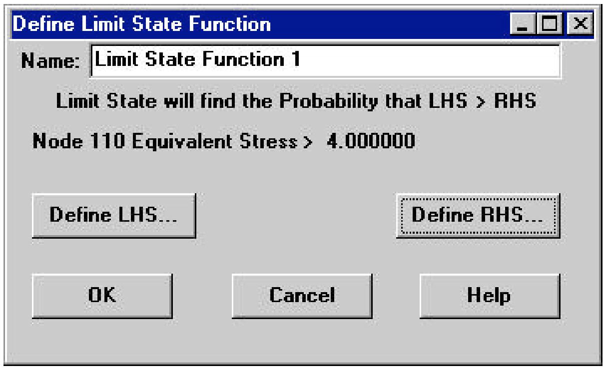

Figure 5. ***ProFES Define Limit State Function Dialog Box. User selects Define LHS and Define RHS to assign response variables or random variables to a limit state function.***

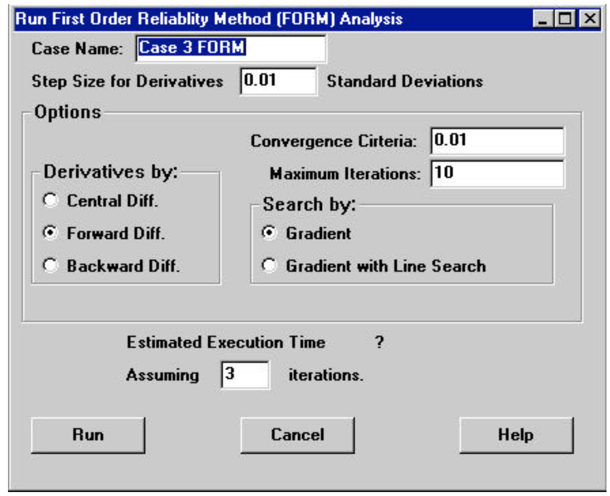

Figure 6. ***ProFES Run First Order Reliability Method (FORM) Analysis Dialog Box. Default values are supplied and are sufficient for most analyses. Advanced users can change the parameters.***

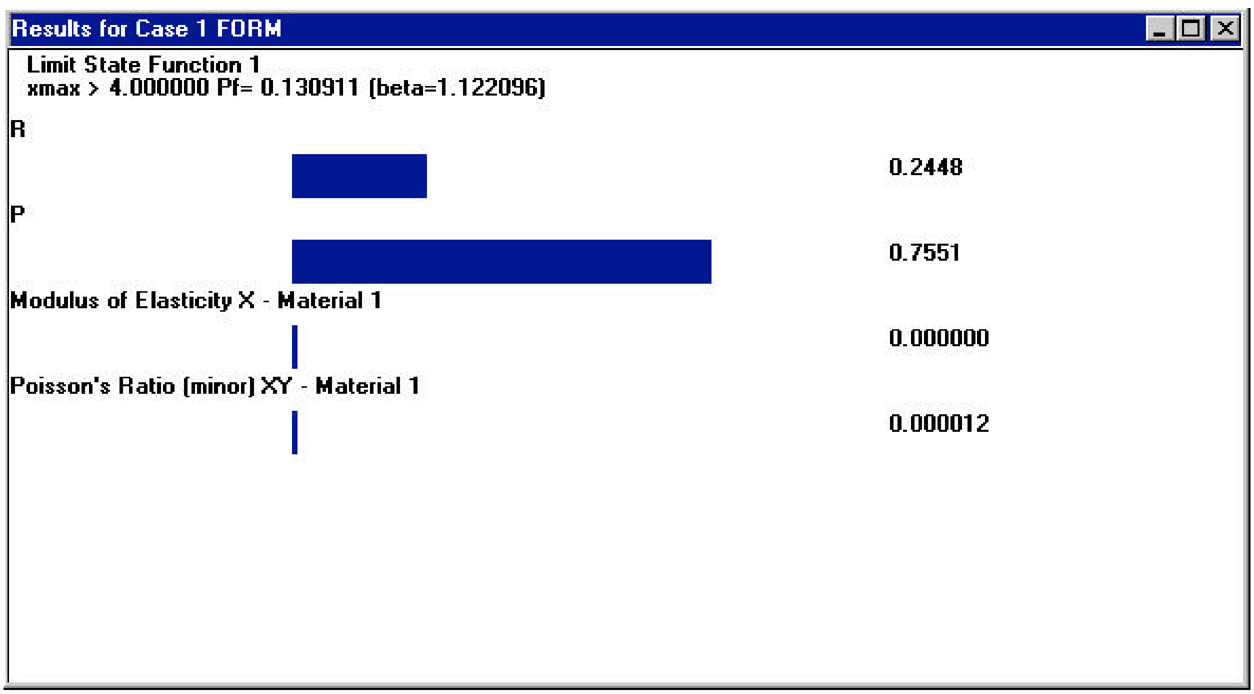

Figure 7. ***FORM analysis results in sensitivity factors for a limit-state defined by the stress concentration being greater than 4.0. The sensitivity results show that the uncertainty is dominated by the uncertainty in the load and the uncertainty in the radius of the hole. These results mean that the product reliability can be improved by tightening the manufacturing tolerance on the radius.***

designs or when using new materials, there may not be sufficient experience to judge what the appropriate safety margin should be. The advantage of the probabilistic or reliability-based approach is that we can redesign to meet a reliability level that is safe and economical.

As mentioned above, Figure 7 also shows the relative contributions of each of the four random variables for this problem. These quantities can be interpreted as the variance in the response attributable to each of the input variables. This information can be useful for redesign, specifying optimal tolerance, or eliminating random variables. For example, the figure shows that along with the load uncertainty the manufacturing variability in the hole radius R is a contributing factor to the probability of failure. Hence, tightening the tolerance will improve the product reliability. Plots like Figure 7 can also be used to identify variables whose tolerances can be relaxed (e.g., if a geometry parameter is shown to have a small contribution), in order to reduce costs.

Example 3 – High Cycle Fatigue on Turbine Blade

Figures 8, 9, and 10 show ProFES solving an HCF (high cycle fatigue) problem for a turbine engine blade. Figure 8 shows the finite element model and nodal loading conditions. This finite element blade model is made of shell elements with a small hollow volume inside. It is fully constrained along the x-axis. A probabilistic material degradation model that accounts for thermal fatigue, creep, and high cycle mechanical fatigue was used to determine failure stress. The degradation model reduces the failure stress, S_0, as shown in Equation 1.[3,5]

$$S = S_0\left(\frac{N_{mn} - N_m}{N_{mn} - N_{m0}}\right)^s\left(\frac{T_{cu} - Tc}{T_{cu} - T_{c0}}\right)^v\left(\frac{N_{Tu} - N_T}{N_{Tu} - N_{T0}}\right)^u \quad \textbf{\textit{(Eq. 1)}}$$

and,

N_m = 250,000 N cycles (mechanical)
T_c = 1,000 N hours
N_T = 2,000 N cycles (thermal)

where,

S_0 = undegraded failure stress
N_{mu} = ultimate mechanical cycles
T_{cu} = ultimate creep hours
N_{tu} = ultimate thermal cycles
N_{m0} = initial value of mechanical cycles
T_{c0} = initial value of creep time
N_{t0} = initial value of thermal cycles
N_m = number of mechanical cycles
T_c = number of creep hours
N_T = number of thermal cycles
N = time
s = degradation exponent for mechanical fatigue
v = degradation exponent for creep
u = degradation exponent for thermal fatigue

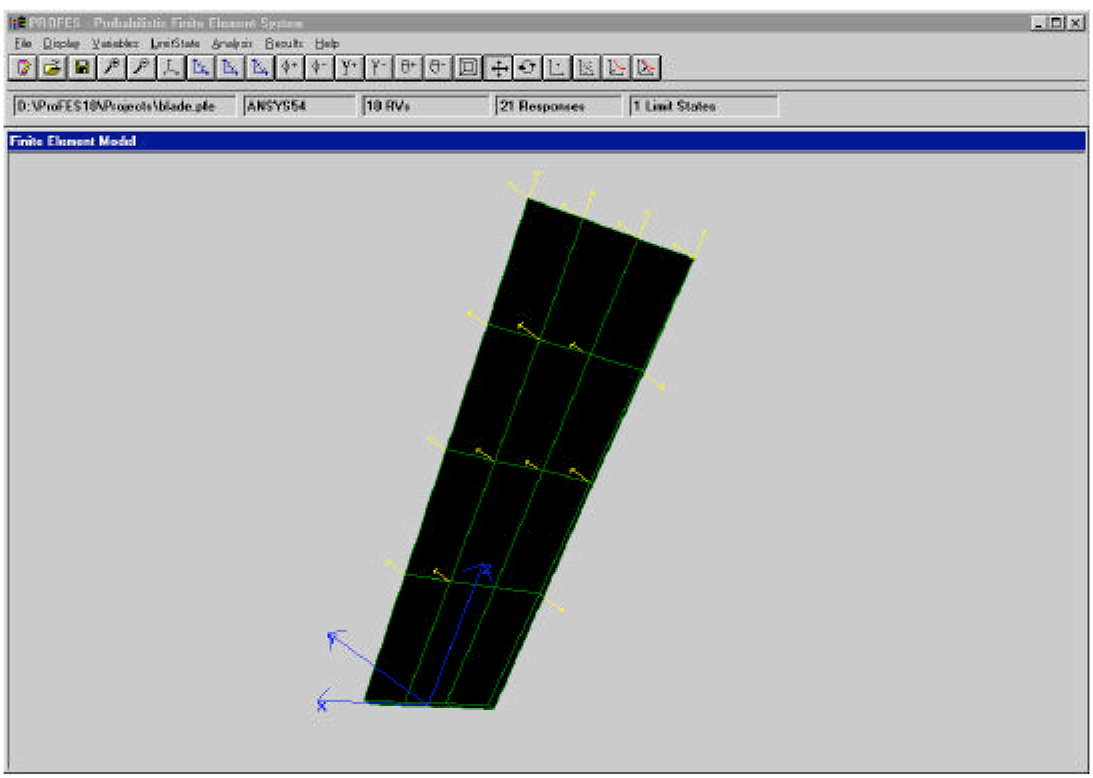

Figure 8. Turbine blade model with applied loads in ProFES viewer window.

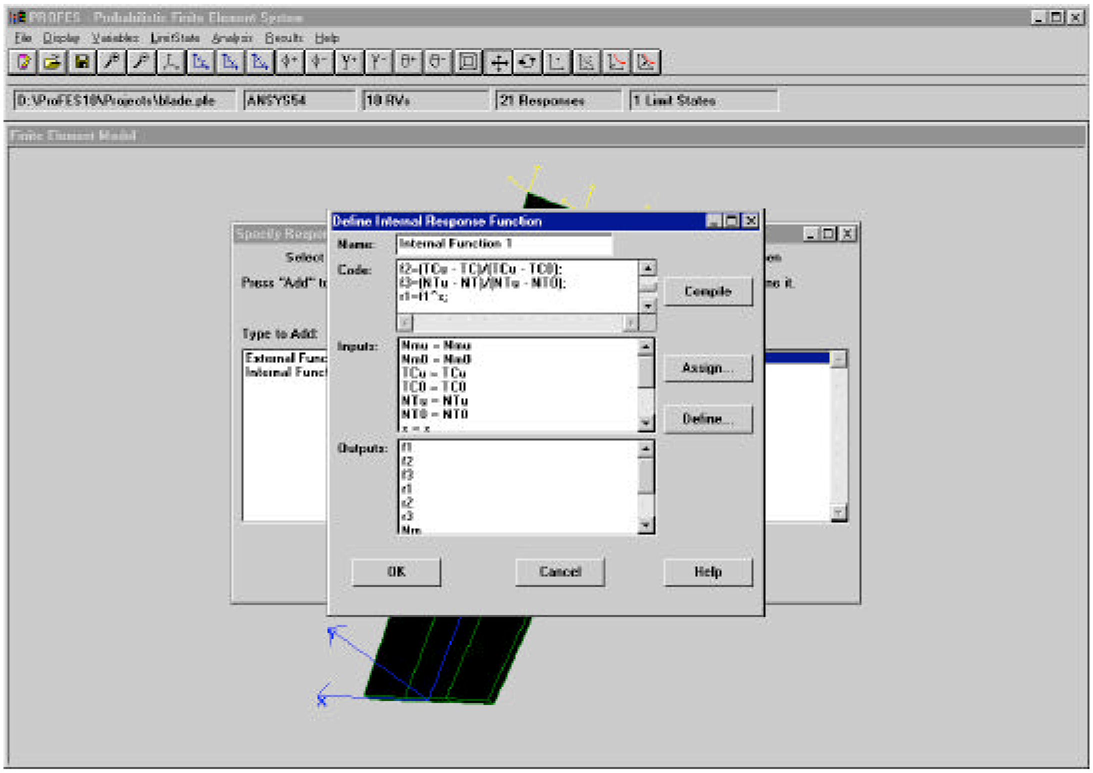

Figure 9. *ProFES Define Internal Function Dialog box used to enter HCF strength degradation equation directly into ProFES.*

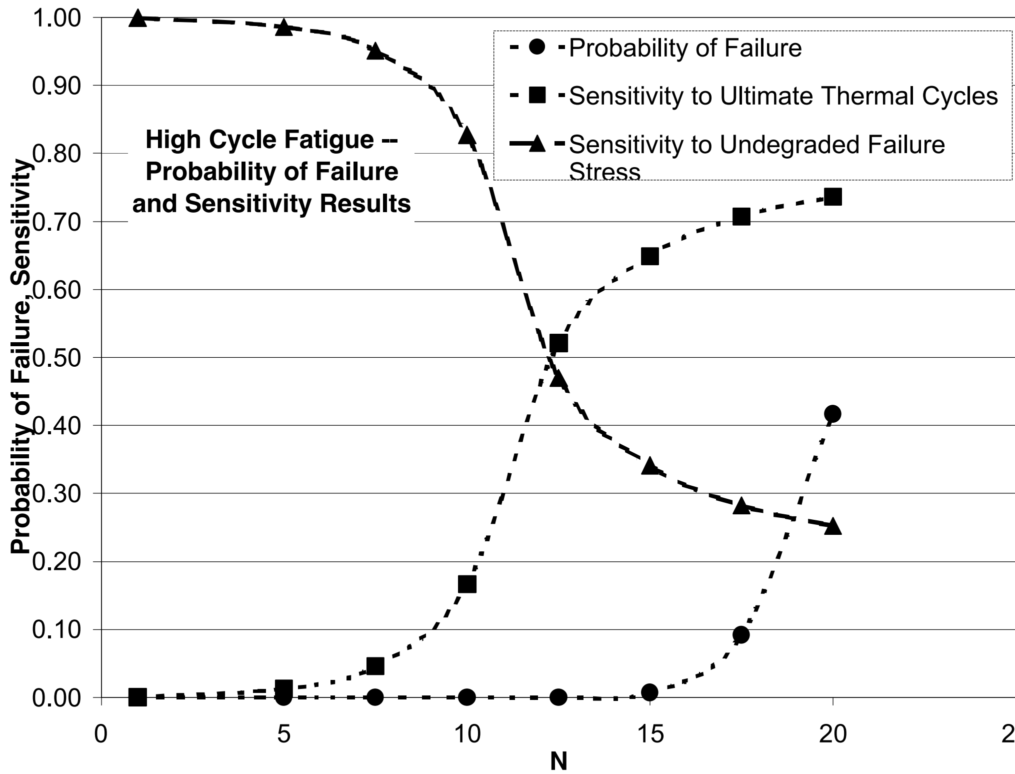

Figure 10. *ProFES was used to generate points for the chart which plots the changes in probability of failure as well as sensitivity to two of the random variables over the life of the component.*

Table 2 shows the random variable distributions for the degradation model parameters in the undamaged failure state. The ProFES internal function feature shown in Figure 9 is used to evaluate this equation. You enters the equation in the *Define Internal Response Function Dialog Box* and presses the *Compile* button. ProFES then determines which variables are inputs and which variables are outputs to this equation. You then assigns finite element results to the variables or defines them to be random variables.

Table 2. HCF example random variable distributions

Parameter	Dist	Mean	Std.Dev.	Units
N_{mu}	Normal	1×10^{10}	1×10^{9}	(cycles)
N_{mo}	Normal	0.25	0.025	(cycles)
T_{cu}	Normal	100,000	10,000	(hours)
T_{co}	Normal	0.25	0.025	(hours)
N_{Tu}	Normal	50,000	5,000	(cycles)
N_{To}	Normal	0.25	0.025	(cycles)
s	Normal	0.2235	0.0067	--
v	Normal	0.1735	0.0052	--
u	Normal	0.1910	0.0057	--
S_o	Lognormal	212,000	10,600	(psi)

A limit state function was then added to determine the probability that the degraded maximum stress is less than the maximum equivalent stress in the finite element model. The current values of N_m, T_c, and N_T were varied by changing N from 1 to 20 to simulate various lifetimes of the component.

As seen in Figure 10, the failure probability (labelled HCF P_f) varies from 0 to 0.42 over this lifetime. Also shown in Figure 10, is the sensitivity factor, α^2, to the undegraded failure stress (S_0) and the ultimate number of thermal cycles (N_{tu}). The random variable sensitivity changes over time from being highly dependent on the undegraded failure stress to being dependent on the ultimate thermal fatigue lifetime.

CONCLUSIONS

ProFES places probabilistic finite element tools in the hands of practitioners and makes them usable without extensive re-training and software development. ProFES includes an innovative data-driven architecture that has the look and feel of commercial CAD and pre-processing packages. The ProFES approach supports transparent interfaces to commercially available finite element packages and allows you to build your deterministic model in your preferred commercial pre-processing package. ProFES has been integrated with several major probabilistic computational methods along with an extensive library of random variable distributions. ProFES also supports easy linkage to external routines for post processing FEA for applications such as probabilistic fatigue life analysis.

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